FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 23 May 2003 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator in the appropriate box on your cover sheet *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

Please start each question on a new page. You are advised to show all working, where possible. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. *[Maximum mark: 17]*

(i) Consider three sets S, T, and U. α and β are two mappings such that $\alpha : S \to T$, and $\beta : T \to U$.

(a) If α and β are injective, prove that $\beta \circ \alpha$ is injective. *[4 marks]*

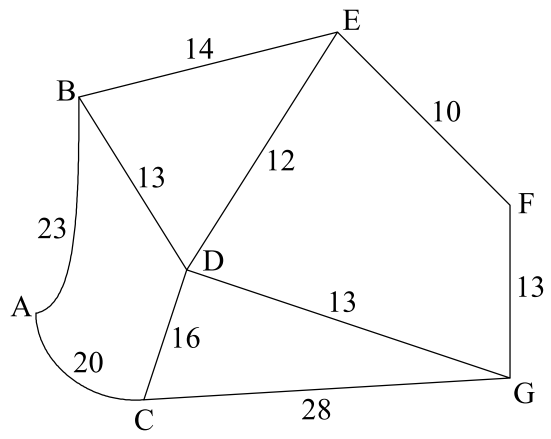
(b) If $\beta \circ \alpha$ is injective, prove that α is injective. *[4 marks]*

(ii) Consider the set $S = \{w, x, y, z\}$ and the operation $*$ defined on its elements. Copy and complete the following table in such a way that $(S, *)$ forms a group. Justify your answer, with reference to group axioms. You may assume that $*$ is associative. *[9 marks]*

$*$	w	x	y	z
w	y			x
x	z	w		
y				
z				w

2. *[Maximum mark: 17]*

(i) The diagram below represents a network.



(a) Starting at E, apply Prim's algorithm to find the minimum spanning tree of this graph. Show all your work clearly, particularly the order in which you added the edges, and the length of the minimum tree. *[7 marks]*

(b) Indicate the differences if you were to apply Kruskal's algorithm instead of Prim's algorithm to this network. *[3 marks]*

(ii) Solve the following recurrence relation

$$a_n - 9a_{n-2} = 0, \text{ with } a_0 = 6 \text{ and } a_2 = 54.$$ *[7 marks]*

3. *[Maximum mark: 21]*

(i) Find the largest interval over which $\sum_{k=0}^{\infty}\left(\frac{3k^2}{e^k}\right)x^k$ will converge. *[8 marks]*

(ii) Consider the converging infinite series $\sum a_k$, where $a_k \geq 0, k \in \mathbb{Z}^+$.

(a) Show that $\sum a_k^2$ converges. *[3 marks]*

(b) (i) Show that $\sum\left(a_k - \frac{1}{k}\right)^2$ converges.

(ii) Hence, show that $\sum \frac{a_k}{k}$ will also converge. *[5 marks]*

(iii) Maclaurin's series is to be used to estimate $e^{0.2}$ with an error term less than 0.0005.

(a) Find an expression for the remainder.

(b) Calculate the number of terms required.

(c) Estimate $e^{0.2}$ correct to **three** decimal places. *[5 marks]*

4. *[Maximum mark: 27]*

(i) A manufacturing plant uses small amounts of iron in its production process. The amount x tons of iron used per month can be modelled by the following probability density function.

$$f(x) = \begin{cases} k\mathrm{e}^{-\frac{x}{4}}, & x \geq 0. \\ 0, & x < 0. \end{cases}$$

(a) Show that $k = \frac{1}{4}$.

(b) Find the probability that the plant uses more than four tons per month.

(c) How much should they stock so that they run out of iron only 5 % of the time? *[8 marks]*

(ii) A comparison of the wearing quality of two types of tyres X and Y was obtained by testing samples of 100 of each type. The number of kilometres before the tyre wears out was recorded. The test results were as follows

$$\bar{x} = 26400 \text{ km} \quad \bar{y} = 25100 \text{ km}$$
$$s_x = 1200 \quad s_y = 1400$$

(a) (i) Estimate the difference in the mean distance before the tyre wears out.

(ii) Find an interval which places a bound of two standard errors on this estimate.

(iii) What level of confidence has been achieved in the above interval?

(b) The producers of the first type X claim that their tyres will outlast the second type, Y, by more than 1000 km. Test the claim at the 5 % level of significance. *[9 marks]*

(This question continues on the following page)

(Question 4 continued)

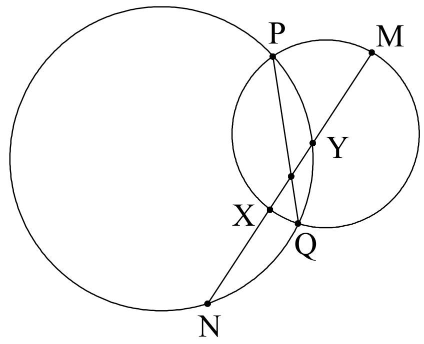
(iii) A scientist uses a microscope to study the number of colonies of bacteria in milk film. The following frequency distribution shows the number of colonies per slide in 400 samples.

Number of colonies	Frequency
0	55
1	104
2	80
3	62
4	42
5	27
6	9
7	9
8	6
9	3
10	2
11	1

Is there sufficient evidence at the 5 % level to claim that the data fits the Poisson distribution? *[10 marks]*

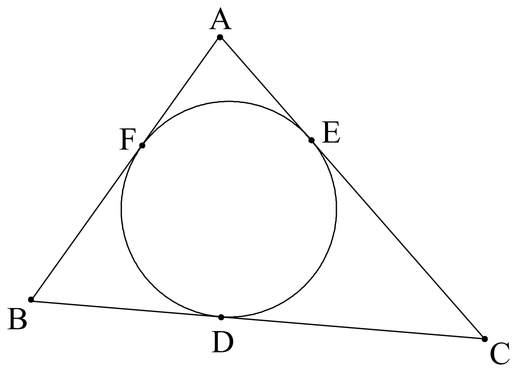
5. *[Maximum mark: 18]*

(i) The diagram below shows two circles with a common chord [PQ]. The points M and N are on the circles such that [PQ] bisects the line segment [MN]. The other two points of intersection of [MN] with the circles are X and Y.



Show that NX = MY. *[4 marks]*

(ii) The diagram shows ΔABC with its incircle. The points of tangency of the circle with the sides [BC], [CA], and [AB] are D, E, and F respectively.



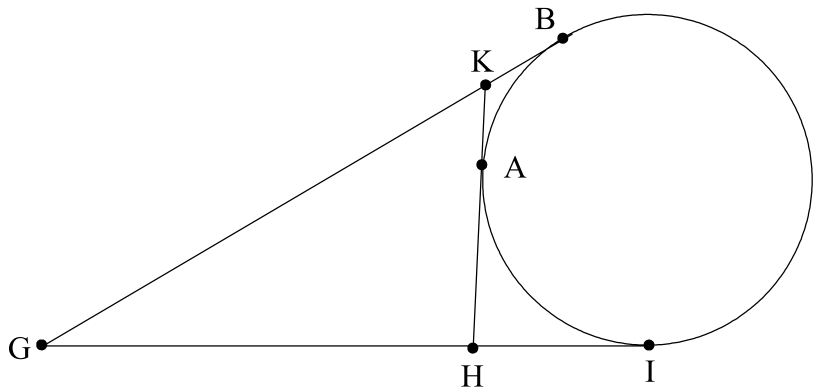
(a) Show that (AD), (BE) and (CF) are concurrent. *[3 marks]*

(b) Given that the point of concurrency found in part (a) is the incentre of the triangle, show that the triangle is equilateral. *[5 marks]*

(This question continues on the following page)

(Question 5 continued)

(iii) In the diagram below [GHI] is a fixed line, with GH = m and HI = n. A circle with variable radius is drawn tangential to (GI) at I. The tangents (GB) and (HA) meet at K.



Find the locus of point K as the radius is varied. Give a complete description of the locus. *[6 marks]*